Exploring the Impact of CAS in Early Algebra.

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This paper reports on an investigation into the impact of the availability of a hand held computer algebra system (CAS) on student performance and patterns of algebraic thinking. In particular, their performance in a range of symbol manipulation tasks in the early stages of the middle school algebra curriculum is discussed.

Introduction

'Every cognitive technology, connected to mathematics in some implicit or explicit way, inevitably influences the practice of mathematics education even if this was not its initial aim (eg language, print, pens and pencils, computers). Pea (1986)

The technology currently available on $250 hand-held symbolic calculators will perform almost all of the tortuous algebraic manipulations found in the secondary school maths curricula in all Australian states. Frequently termed a Computer Algebra System (CAS), this 'cognitive technology' raises questions about the mathematical experiences we believe our students should have. For example, what symbolic manipulations should our students be able to complete by hand?

To illustrate the immediacy of such questions, consider the following situation, which arose last year in a Year 12 mathematics class in Victoria. A teacher refused to authenticate a student's work in a major Year 12 mathematics assessment task, because the student had used a CAS to do algebra manipulations which he could not do himself (Roberts, 1997a). In a follow up to the original article (Roberts, 1997b), the teacher added the following comments;

'we did give him credit for correctly formulating the equations - he was intellectually very gifted, but was also very lazy, which was partly why his algebra was weak (he didn't practice, and was very careless/untidy)'

Kutzler (1997) contends that when students perform an algebraic procedure such as equation solving, it is possible to distinguish two types of steps in the solution process: *choosing* - deciding the operation to be performed and *applying* - carrying out that operation.

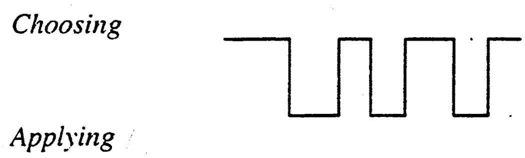

Figure 1

Kutzler argues that in most algebraic procedures, the higher level task of 'choosing' is repeatedly interrupted by the lower level task of 'applying' (see Figure 1). He suggests that a CAS allows the student to concentrate on the higher level tasks of choosing and responding to the feedback given by each choice.

So should the work of Roberts' student have been authenticated? It appears that he may have known what steps needed to be performed, but could not complete them without the aid of CAS. In response to Roberts' article, Stacey suggested a newer set of criteria for assessing a students 'symbol sense' (Stacey, 1997). Students might be said to understand a procedure if:

a) they know what the procedure is for;
b) they know what sort of input is required;
c) they know under what conditions the procedure can be used;
d) they can make sense of the answer that the software produces;
e) they have some capacity to detect errors in the answers given by the software (sufficient to guard against input errors, for example).

These criteria attempt to provide checks that the student and not the technology is at the helm of a symbolic procedure. It is critical to identify, in a technology rich environment, which cognitive tasks should be retained by the individual and which tasks should be 'distributed' to the technology.

Determining the importance of 'symbol sense' may be guided by what has happened with the impact of the four-function calculator on arithmetic. We realised early that it made sense not to teach some things any more. Long division especially with large numbers and the use of logarithm tables and their many complicated variations left the mainstream curriculum. We also soon came to see that children could not use calculators sensibly if they did not have a good sense of number — button pushing is prone to silly mistakes, so knowing roughly what the answer will be like is essential (Groves, 1994). We now need to discover what experiences our students require in order to develop solid symbol sense, and how to create an environment where CAS may be both accessible and affordable for the majority.

Very little research has been conducted into the impact of CAS in the secondary school curriculum, and even less at the middle grades. The main thrust of the studies that have been undertaken is on the effects of replacing or de-emphasising by-hand symbol manipulations with computer generated symbol manipulations in post-secondary 'reform' calculus courses. Keller and Russell (1997) reports that five of seven semester long investigations found improved understanding of calculus concepts with no loss of computational skill. The other two studies found no difference in performance on departmental final exams. In a study incorporating CAS use in a quadratic functions unit with 14-15 year old students, Hunter commented that a CAS can be of benefit for the student's learning of abstract elements of algebra as long as the students were mathematically ready to use it (Hunter et al, 1995).

How might the classroom environment change in a CAS enriched environment? Clearly, a CAS can be used to perform and check by-hand calculations, and do this quicker and more effectively than we have been able to do in the past. In this sense, we are using the technology to 'amplify' the current curriculum. Alternatively, by holding up a mirror to current classroom practise, CAS may also act as a 'reorganiser' in the curriculum. Dorfler (in Hillel, 1993) argues that technology changes the nature of an activity or task in which it is employed in the following ways:

- stronger differentiation between planning & execution of mathematical algorithms/operations;
- requires more detailed planning/anticipation of results;
- change in nature of objects to operate on (eg complicated functions);

- change in representation of objects and processes (eg notation changes, functional representations, geometric objects);
- changes to sequencing of topics.

This study attempts to explore these issues in the context of the algebra curriculum.

Methodology

A teaching experiment was conducted to explore the effects of student use of a hand-held computer algebra system (a Texas Instruments TI-92) on their understandings of key algebra concepts and procedures related to these tasks. The tasks focussed on:

- symbol manipulation techniques;
- formulation of expressions and equations from word statements;
- solving problems which involve both formulation and symbol manipulation techniques.

The experiment was conducted in one single sex school and involved two classes of female year 9 students, taught by two teachers. One class was assigned as the experimental group and were given TI-92s for the duration of the unit. The other class became the control group and had access to Texas Instruments TI-82s. Each class was taught a unit on linear functions.

Comparisons were drawn between the CAS group and the non-CAS group in relation to the three task foci. In addition, because little is known at present about the use of a CAS at the high school level, general issues concerning its use in the teaching and learning of mathematics at this level were explored and documented.

The learning outcomes were negotiated and developed by the two teachers and the researchers to ensure that outcomes included some focus on the three key aspects identified above: symbol manipulation, formulation, and problem solving. Pre and post tests for the unit based on the agreed learning outcomes were developed. Results from pre-tests were used to determine the comparability of classes and act as a covariate for analysis of treatment effects.

The post tests without use of the CAS were used to measure the immediate effects of CAS use on student understandings and ability to perform paper and pencil tasks related to symbol manipulation techniques, equation formulation and problem solving. A delayed post test is planned for the second term of 1998. Those classes taught with CAS were given an additional CAS 'active' test which permits the use of CAS. Results from the tests provided information on student ability to use the technology, on student ability to adapt to the new notational system required for operation of the CAS, and on general problems and difficulties experienced. Teachers and students were asked to keep a journal of classroom observations and their reflections on these observations and general issues of CAS classroom use. In addition, the research team made their own classroom observations of classes.

Results and Discussion

Task types and CAS use

In skill mastery tasks throughout the instructional unit, the CAS symbol manipulator provided an additional means of student 'checking' their answers. Many students commented that they found it very useful to have an electronic 'answer page', and there was evidence from student journal comments to suggest that this provided increased motivation for completing by-hand manipulation exercises within the unit.

Post-test results suggest that students in the CAS group were more inclined to persist with algebraic methods (usually by performing the same operation on both sides of an equation) for equation solving tasks, particularly when the level of difficulty of the manipulations increased. This was true even for test items where CAS use was not available (Table 1). It was also noted that the 'backtracking' method was more prevalent in the non-CAS group post-test results.

Sample Problem 1 - Solve for m: $\frac{3m+2}{5} = -4m$

Method	CAS Group	Non-CAS Group
Algebraic (successful)	*7*	*0*
Algebraic (unsuccessful)	*12*	*6*
Backtracking (successful)	*0*	*0*
Backtracking (unsuccessful)	*0*	*5*
No Attempt	*6*	*12*
	25	*23*

Table 1 - Student Response Summary for Sample Problem 1

In the post-test tasks requiring formulation (where no CAS was permitted), the CAS group were also more likely to use algebra than arithmetic, that is, attempting to construct and then solve an equation rather than by using backtracking or 'Trial and Error' (Table 2). However, when their results were analysed further, students from the CAS group were no more successful in completing such tasks than students from the non-CAS group.

Sample Problem 2 - I think of a number, multiply it by 5, subtract 3 and divide by 2. The result is 4 more than twice than the number I first thought of. What was the number?

Method	CAS Group	Non-CAS Group
Algebra	*14*	*6*
Backtracking	*1*	*4*
Trial and Error	*4*	*5*
No attempt	*4*	*8*
	25	*23*

Table 2 - Student Response Summary for Sample Problem 2

There was a clear trend in the types of errors students from both groups made in equation solving tasks, with mistakes commonly due to poor order of operations skills, and an inability to deal with negative number operations. None of these mistakes arose when students chose to use the CAS to perform such tasks.

In CAS 'active' post-test questions requiring students to describe each step performed in an equation solving task (CAS group only), students who chose to use the CAS were spectacularly successful, while those who chose not to use the tool often did not complete this task well (Table 3). It appeared that the CAS symbol manipulator was effective in focussing attention on the effects of manipulating symbols in an equation, and in formalising the approach used.

Sample Problem 3 - Use your TI-92 to solve the following equations, and fill in the table below to show what operations you have performed.

(a) Solve this equation for x: $\frac{4(x-7)}{3} = 3 - x$		
Operation	Performing operation	After Operation

Method	*Correct*	*Incorrect*	*TOTAL*
CAS used	*12*	*1*	*13*
CAS not used	*2*	*10*	*12*

Table 3 - Student Response Summary for Sample Problem 3

There were instances where students using the CAS-assisted approach would use an unconventional sequence of operations to solve the equations. In some cases this resulted in extra steps, but the students were still successful in completing the task. In contrast, students who did not use CAS were able to choose appropriate operations but were often unable to complete the manipulations. This result echoes findings by Coady and Pegg (1995) who, in attempting to identify the levels of higher order algebraic skills possessed by undergraduate mathematics students, noted that one major difference between student responses was in the 'ability to form a plan for the entire set of procedures required'. They noted that students often were able to begin procedures, but made manipulation errors or chose procedures that were unproductive.

Manipulation skills

There is a fear that frequent CAS use will reduce student's by-hand algebra skills.

> *'techniques and tedious calculations ... are as rigged and as tedious as finger exercises for the pianist, and just as indispensable'* Halmos 1990

In this study, the teachers expected that students in the CAS group would be able to perform the same by-hand manipulations as the students from the non-CAS group. The post-test results showed no significant differences between the groups (Table 4). This result mirrors the majority of studies where a CAS has been introduced.

	CAS Group	**Non-CAS Group**
Substitution/Expansion/Factorisation	8.0	7.2
Equation Formulation	1.2	0.8
Equation Solving Skills	6.7	6.8
Overall Post-test	15.6	15.2

Table 4 - Comparison of post-test mean scores (adjusted by the pretest covariate)

Rather than an over-reliance on the CAS, evidence from student journals, classroom observations, and teacher interviews suggest that under-use was more prevalent. Factors contributing to this include:

- some better students believed that the CAS was more tedious or cumbersome to use for the problems at hand.

  "I thought that the calculator was a waste of time because I already knew how to do everything by hand."
- some students believed that the CAS was like 'cheating', and felt that it was not beneficial for them to rely on it. That is, they had short term access only, and the CAS in question was banned for Year 12 examination purposes.
- some students expressed frustration with the notation system or hardware/software.

  "I can do these in my head and get them right according to the book (text) but on the calculator I get them (some) wrong."
- some students were less able and/or poorly motivated in mathematics classes, and saw the CAS as an additional burden. This was reflected in test results and student journals.

  "To [Teacher Name],
  [Student Name] tried to work the super gadget, and got nowhere, I tried after, then her brother. None of us could make sense of it."

  (signed by parent)
- teacher beliefs impacted how enthusiastically students took up calculator use. The calculator was only available to students during the study and therefore the teachers felt it was unwise for students to become dependent on it.

Notation issues

Mathematics has a complex set of notations in common use. The school curriculum has until recently carefully controlled the notations which students use at each stage of schooling. This changes with the use of CAS technology. A CAS uses symbols that they would not normally be exposed to in Year 8-10 mathematics, and in this unit students were introduced to notation that has a number of critical differences to the traditional notation. Some examples of syntax used by students in this unit are given in Figure 2.

solve(3m+2)/5=-4m,m)   *(3m+2)/5 | m=3*   *factor(2a+2b=7)*

Figure 2

Students were occasionally frustrated by the error messages which appeared. For example, a student entered **expand(b(b-2))** and the calculator returned the error message "Invalid implied multiply". Other students commented on how the calculator's auto-simplify feature would alter the form of the expressions or equations that they had entered

"After completing the sheet, I find it really annoying the way the calculator changes what we type in, into its own little order"

Our experience with the Technology Enriched Algebra project (Tynan et al. 1995) and observations from this study suggest that students are more adaptable with notation than we had thought, and indeed by the end of the study period most students were aware of the meaning of error messages and had adapted to the notational demands of the calculator (at least as far as the content of this unit was concerned).

CAS and Curriculum Change

Some of the tasks in the unit were designed to make best use of the technology's 'what-if' features. These tasks support inquiry-based teaching approaches, rather than direct instructional methods. We found instances where the style of the tasks conflicted with the task type to which the students and teacher were familiar (see Artigue 1996 for further discussion of this issue). Tasks requiring open-ended student investigation were in some cases less successful than those tasks requiring a more limited but repetitive student response.

It may well be that the sort of curriculum change in which CAS becomes an indispensable tool will be a slow evolution rather than a revolution (Mann, Rothery & Sato 1997). Our experiences in this study lead us to believe there are a large number of factors that can affect the pedagogic impact of CAS at this level:

- the amount of student access to tool;
- teacher knowledge, beliefs and preferred style;
- student algebraic skills and preferred learning style;
- student motivation to succeed/persist;
- structural factors - competing pressures on student/teacher time;
- level of communication between research team and teachers;
- level of support for teachers;
- task type design - what interaction is encouraged between user and CAS;
- task type balance open-ended and more routine tasks.

Conclusion

CAS did appear to have some impact on student preferred methods of equation solving, and there was evidence that CAS helped students focus on choosing appropriate manipulations. Most students were enthusiastic about using CAS as a dynamic checking device, and post-test results indicate CAS use did not reduce student's by-hand algebra skills.

It is clear that the demands of the current courses, and the need to at least cover all of the existing curriculum material as well as more CAS-active tasks will continue to be a problem for teachers, and further work is needed to identify the tasks, strategies and balance of by-hand and CAS work appropriate at this level. The pedagogic impact of CAS at this level will be affected by many variables other than the presence of the technology itself, and attempts to control all these variables in any teaching experiment are at best problematic.

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